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# The Stochastic Growth Model

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BusinessSumup

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# Contents

# 1. Introduction

This article presents the stochastic growth model. The stochastic growth model is a stochastic version of the neoclassical growth model with microfoundations,[1] and provides the backbone of a lot of macroeconomic models that are used in modern macroeconomic research. The most popular way to solve the stochastic growth model, is to linearize the model around a steady state,[2] and to solve the linearized model with the method of undetermined coefficients. This solution method is due to Campbell (1994).

The set-up of the stochastic growth model is given in the next section. Section 3 solves for the steady state, around which the model is linearized in section 4. The linearized model is then solved in section 5. Section 6 shows how the economy responds to stochastic shocks. Some concluding remarks are given in section 7.

# 2. The stochastic growth model

**The representative firm** Assume that the production side of the economy is represented by a representative firm, which produces output according to a Cobb-Douglas production function:

$$Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha} \qquad \text{with } 0 < \alpha < 1 \tag{1}$$

$Y$ is aggregate output, $K$ is the aggregate capital stock, $L$ is aggregate labor supply and $A$ is a technology parameter. The subscript $t$ denotes the time period.

The aggregate capital stock depends on aggregate investment $I$ and the depreciation rate $\delta$:

$$K_{t+1} = (1-\delta)K_t + I_t \qquad \text{with } 0 \leq \delta \leq 1 \tag{2}$$

The productivity parameter $A$ follows a stochastic path with trend growth $g$ and an AR(1) stochastic component:

$$\begin{aligned} \ln A_t &= \ln A_t^* + \hat{A}_t \\ \hat{A}_t &= \phi_A \hat{A}_{t-1} + \varepsilon_{A,t} \qquad \text{with } |\phi_A| < 1 \\ A_t^* &= A_{t-1}^*(1+g) \end{aligned} \tag{3}$$

The stochastic shock $\varepsilon_{A,t}$ is i.i.d. with mean zero.

The goods market always clears, such that the firm always sells its total production. Taking current and future factor prices as given, the firm hires labor and invests in its capital stock to maximize its current value. This leads to the following first-order-conditions:[3]

$$(1-\alpha)\frac{Y_t}{L_t} = w_t \tag{4}$$

$$1 = E_t\left[\frac{1}{1+r_{t+1}}\alpha\frac{Y_{t+1}}{K_{t+1}}\right] + E_t\left[\frac{1-\delta}{1+r_{t+1}}\right] \tag{5}$$

According to equation (4), the firm hires labor until the marginal product of labor is equal to its marginal cost (which is the real wage $w$). Equation (5) shows that the firm's investment demand at time $t$ is such that the marginal cost of investment, 1, is equal to the expected discounted marginal product of capital at time $t+1$ plus the expected discounted value of the extra capital stock which is left after depreciation at time $t+1$.

**The government** The government consumes every period $t$ an amount $G_t$, which follows a stochastic path with trend growth $g$ and an AR(1) stochastic component:

$$
\begin{aligned}
\ln G_t &= \ln G_t^* + \hat{G}_t \\
\hat{G}_t &= \phi_G \hat{G}_{t-1} + \varepsilon_{G,t} \qquad \text{with } |\phi_G| < 1 \qquad (6) \\
G_t^* &= G_{t-1}^*(1+g)
\end{aligned}
$$

The stochastic shock $\varepsilon_{G,t}$ is i.i.d. with mean zero. $\varepsilon_A$ and $\varepsilon_G$ are uncorrelated at all leads and lags. The government finances its consumption by issuing public debt, subject to a transversality condition,[4] and by raising lump-sum taxes.[5] The timing of taxation is irrelevant because of Ricardian Equivalence.[6]

**The representative household** There is one representative household, who derives utility from her current and future consumption:

$$U_t = E_t\left[\sum_{s=t}^{\infty}\left(\frac{1}{1+\rho}\right)^{s-t}\ln C_s\right] \qquad \text{with } \rho > 0 \tag{7}$$

The parameter $\rho$ is called the subjective discount rate.

Every period $s$, the household starts off with her assets $X_s$ and receives interest payments $X_s r_s$. She also supplies $L$ units of labor to the representative firm, and therefore receives labor income $w_s L$. Tax payments are lump-sum and amount to $T_s$. She then decides how much she consumes, and how much assets she will hold in her portfolio until period $s+1$. This leads to her *dynamic budget constraint*:

$$X_{s+1} = X_s(1+r_s) + w_s L - T_s - C_s \tag{8}$$

We need to make sure that the household does not incur ever increasing debts, which she will never be able to pay back anymore. Under plausible assumptions, this implies that over an infinitely long horizon the present discounted value of the household's assets must be zero:

$$\lim_{s\to\infty} E_t\left[\left(\prod_{s'=t}^{s}\frac{1}{1+r_{s'}}\right)X_{s+1}\right] = 0 \tag{9}$$

This equation is called the *transversality condition*.

The household then takes $X_t$ and the current and expected values of $r$, $w$, and $T$ as given, and chooses her consumption path to maximize her utility (7) subject to her dynamic budget constraint (8) and the transversality condition (9). This leads to the following Euler equation:[7]

$$\frac{1}{C_s} = E_s\left[\frac{1+r_{s+1}}{1+\rho}\frac{1}{C_{s+1}}\right] \tag{10}$$

**Equilibrium** Every period, the factor markets and the goods market clear. For the labor market, we already implicitly assumed this by using the same notation ($L$) for the representative household's labor supply and the representative firm's labor demand. Equilibrium in the goods market requires that

$$Y_t = C_t + I_t + G_t \tag{11}$$

Equilibrium in the capital market follows then from Walras' law.

# 3. The steady state

Let us now derive the model's balanced growth path (or *steady state*); variables evaluated on the balanced growth path are denoted by a $^*$.

To derive the balanced growth path, we assume that by sheer luck $\varepsilon_{A,t} = \hat{A}_t = \varepsilon_{G,t} = \hat{G}_t = 0$, $\forall t$. The model then becomes a standard neoclassical growth model, for which the solution is given by:[8]

$$
\begin{aligned}
Y_t^* &= \left(\frac{\alpha}{r^* + \delta}\right)^{\frac{\alpha}{1-\alpha}} A_t^* L && (12)\\
K_t^* &= \left(\frac{\alpha}{r^* + \delta}\right)^{\frac{1}{1-\alpha}} A_t^* L && (13)\\
I_t^* &= (g+\delta)\left(\frac{\alpha}{r^* + \delta}\right)^{\frac{1}{1-\alpha}} A_t^* L && (14)\\
C_t^* &= \left[1 - (g+\delta)\frac{\alpha}{r^* + \delta}\right]\left(\frac{\alpha}{r^* + \delta}\right)^{\frac{\alpha}{1-\alpha}} A_t^* L - G_t^* && (15)\\
w_t^* &= (1-\alpha)\left(\frac{\alpha}{r^* + \delta}\right)^{\frac{\alpha}{1-\alpha}} A_t^* && (16)\\
r^* &= (1+\rho)(1+g) - 1 && (17)
\end{aligned}
$$

# 4. Linearization around the balanced growth path

Let us now linearize the model presented in section 2 around the balanced growth path derived in section 3. Loglinear deviations from the balanced growth path are denoted by a ˆ (so that $\hat{X} = \ln X - \ln X^*$).

Below are the loglinearized versions of the production function (1), the law of motion of the capital stock (2), the first-order conditions (4) and (5), the Euler equation (10) and the equilibrium condition (11):[9]

$$\hat{Y}_t = \alpha \hat{K}_t + (1-\alpha)\hat{A}_t \tag{18}$$

$$\hat{K}_{t+1} = \frac{1-\delta}{1+g}\hat{K}_t + \frac{g+\delta}{1+g}\hat{I}_t \tag{19}$$

$$\hat{Y}_t = \hat{w}_t \tag{20}$$

$$E_t\left[\frac{r_{t+1}-r^*}{1+r^*}\right] = \frac{r^*+\delta}{1+r^*}\left[E_t(\hat{Y}_{t+1}) - E_t(\hat{K}_{t+1})\right] \tag{21}$$

$$\hat{C}_t = E_t\left[\hat{C}_{t+1}\right] - E_t\left[\frac{r_{t+1}-r^*}{1+r^*}\right] \tag{22}$$

$$\hat{Y}_t = \frac{C_t^*}{Y_t^*}\hat{C}_t + \frac{I_t^*}{Y_t^*}\hat{I}_t + \frac{G_t^*}{Y_t^*}\hat{G}_t \tag{23}$$

The loglinearized laws of motion of $A$ and $G$ are given by equations (3) and (6):

$$\hat{A}_{t+1} = \phi_A \hat{A}_t + \varepsilon_{A,t+1} \tag{24}$$

$$\hat{G}_{t+1} = \phi_G \hat{G}_t + \varepsilon_{G,t+1} \tag{25}$$

# 5. Solution of the linearized model

I now solve the linearized model, which is described by equations (18) until (25).

First note that $\hat{K}_t$, $\hat{A}_t$ and $\hat{G}_t$ are known in the beginning of period $t$: $\hat{K}_t$ depends on past investment decisions, and $\hat{A}_t$ and $\hat{G}_t$ are determined by current and past values of respectively $\varepsilon_A$ and $\varepsilon_G$ (which are exogenous). $\hat{K}_t$, $\hat{A}_t$ and $\hat{G}_t$ are therefore called period $t$'s *state variables*. The values of the other variables in period $t$ are endogenous, however: investment and consumption are chosen by the representative firm and the representative household in such a way that they maximize their profits and utility ($\hat{I}_t$ and $\hat{C}_t$ are therefore called period $t$'s *control variables*); the values of the interest rate and the wage are such that they clear the capital and the labor market.

Solving the model requires that we express period $t$'s endogenous variables as functions of period $t$'s state variables. The solution of $\hat{C}_t$, for instance, therefore looks as follows:

$$\hat{C}_t = \varphi_{CK}\hat{K}_t + \varphi_{CA}\hat{A}_t + \varphi_{CG}\hat{G}_t \tag{26}$$

The challenge now is to determine the $\varphi$-coefficients.

First substitute equation (26) in the Euler equation (22):

$$\begin{aligned} &\varphi_{CK}\hat{K}_t + \varphi_{CA}\hat{A}_t + \varphi_{CG}\hat{G}_t \\ &\qquad = E_t\left[\varphi_{CK}\hat{K}_{t+1} + \varphi_{CA}\hat{A}_{t+1} + \varphi_{CG}\hat{G}_{t+1}\right] - E_t\left[\frac{r_{t+1}-r^*}{1+r^*}\right] \end{aligned} \tag{27}$$

Now eliminate $E_t[(r_{t+1}-r^*)/(1+r^*)]$ with equation (21), and use equations (18), (24) and (25) to eliminate $\hat{Y}_{t+1}$, $\hat{A}_{t+1}$ and $\hat{G}_{t+1}$ in the resulting expression. This leads to a relation between period $t$'s state variables, the $\varphi$-coefficients and $\hat{K}_{t+1}$:

$$\begin{aligned}
&\varphi_{CK}\hat{K}_t + \varphi_{CA}\hat{A}_t + \varphi_{CG}\hat{G}_t \\
&\quad = \left(\varphi_{CK} + (1-\alpha)\frac{r^* + \delta}{1+r^*}\right)\hat{K}_{t+1} + \left(\varphi_{CA} - (1-\alpha)\frac{r^* + \delta}{1+r^*}\right)\phi_A\hat{A}_t + \varphi_{CG}\phi_G\hat{G}_t
\end{aligned} \tag{28}$$

We now derive a second relation between period $t$'s state variables, the $\varphi$-coefficients and $\hat{K}_{t+1}$: rewrite the law of motion (19) by eliminating $\hat{I}_t$ with equation (23); eliminate $\hat{Y}_t$ and $\hat{C}_t$ in the resulting equation with the production function (18) and expression (26); note that $I^* = K^*(g+\delta)$; and note that $(1-\delta)/(1+g) + (\alpha Y_t^*)/(K_t^*(1+g)) = (1+r^*)/(1+g)$. This yields:

$$\begin{aligned}
\hat{K}_{t+1} = &\left[\frac{1+r^*}{1+g} - \frac{C^*}{K^*(1+g)}\varphi_{CK}\right]\hat{K}_t \\
&+ \left[\frac{(1-\alpha)Y^*}{K^*(1+g)} - \frac{C^*}{K^*(1+g)}\varphi_{CA}\right]\hat{A}_t - \left[\frac{G^*}{K^*(1+g)} + \frac{C^*}{K^*(1+g)}\varphi_{CG}\right]\hat{G}_t
\end{aligned} \tag{29}$$

Substituting equation (29) in equation (28) to eliminate $\hat{K}_{t+1}$ yields:

$$\begin{aligned}
&\varphi_{CK}\hat{K}_t + \varphi_{CA}\hat{A}_t + \varphi_{CG}\hat{G}_t \\
&\quad = \left[\varphi_{CK} + (1-\alpha)\frac{r^* + \delta}{1+r^*}\right]\left[\frac{1+r^*}{1+g} - \frac{C^*}{K^*(1+g)}\varphi_{CK}\right]\hat{K}_t \\
&\qquad + \left[\varphi_{CK} + (1-\alpha)\frac{r^* + \delta}{1+r^*}\right]\left[\frac{(1-\alpha)Y^*}{K^*(1+g)} - \frac{C^*}{K^*(1+g)}\varphi_{CA}\right]\hat{A}_t \\
&\qquad - \left[\varphi_{CK} + (1-\alpha)\frac{r^* + \delta}{1+r^*}\right]\left[\frac{G^*}{K^*(1+g)} + \frac{C^*}{K^*(1+g)}\varphi_{CG}\right]\hat{G}_t \\
&\qquad + \left(\varphi_{CA} - (1-\alpha)\frac{r^* + \delta}{1+r^*}\right)\phi_A\hat{A}_t - \varphi_{CG}\phi_G\hat{G}_t
\end{aligned} \tag{30}$$

As this equation must hold for all values of $\hat{K}_t$, $\hat{A}_t$ and $\hat{G}_t$, we find the following system of three equations and three unknowns:

$$\varphi_{CK} = \left[\varphi_{CK} + (1-\alpha)\frac{r^* + \delta}{1+r^*}\right]\left[\frac{1+r^*}{1+g} - \frac{C^*}{K^*(1+g)}\varphi_{CK}\right] \tag{31}$$

$$\begin{aligned}
\varphi_{CA} = &\left[\varphi_{CK} + (1-\alpha)\frac{r^* + \delta}{1+r^*}\right]\left[\frac{(1-\alpha)Y^*}{K^*(1+g)} - \frac{C^*}{K^*(1+g)}\varphi_{CA}\right] \\
&+ \left(\varphi_{CA} - (1-\alpha)\frac{r^* + \delta}{1+r^*}\right)\phi_A
\end{aligned} \tag{32}$$

$$\varphi_{CG} = -\left[\varphi_{CK} + (1-\alpha)\frac{r^* + \delta}{1+r^*}\right]\left[\frac{G^*}{K^*(1+g)} + \frac{C^*}{K^*(1+g)}\varphi_{CG}\right] - \varphi_{CG}\phi_G \tag{33}$$

Now note that equation (31) is quadratic in $\varphi_{CK}$:

$$Q_0 + Q_1\varphi_{CK} + Q_2\varphi_{CK}^2 \;=\; 0 \tag{34}$$

where $Q_0 = -(1-\alpha)\frac{r^*+\delta}{1+g}$, $Q_1 = (1-\alpha)\frac{r^*+\delta}{1+r^*}\frac{C_t^*}{K_t^*(1+g)} - \frac{r^*-g}{1+g}$ and $Q_2 = \frac{C_t^*}{K_t^*(1+g)}$

This quadratic equation has two solutions:

$$\varphi_{CK1,2} \;=\; \frac{-Q_1 \pm \sqrt{Q_1^2 - 4Q_0Q_2}}{2Q_2} \tag{35}$$

It turns out that one of these two solutions yields a *stable* dynamic system, while the other one yields an *unstable* dynamic system. This can be recognized as follows.

Recall that there are three state variables in this economy: $K$, $A$ and $G$. $A$ and $G$ may undergo shocks that pull them away from their steady states, but as $|\phi_A|$ and $|\phi_G|$ are less than one, equations (3) and (6) imply that they are always expected to converge back to their steady state values. Let us now look at the expected time path for $K$, which is described by equation (29). If $K$ is not at its steady state value (i.e. if $\hat{K} \neq 0$), $K$ is expected to converge back to its steady state value if the absolute value of the coefficient of $\hat{K}_t$ in equation (29), $\frac{1+r^*}{1+g} - \frac{C^*}{K^*(1+g)}\varphi_{CK}$, is less than one; if $|\frac{1+r^*}{1+g} - \frac{C^*}{K^*(1+g)}\varphi_{CK}| > 1$, $\hat{K}$ is expected to increase - which means that $K$ is expected to run away along an explosive path, ever further away from its steady state.

Let us therefore evaluate the coefficient $\frac{1+r^*}{1+g} - \frac{C^*}{K^*(1+g)}\varphi_{CK}$, which we call $\varphi_{KK}$. Rewriting yields $\varphi_{CK} = \frac{K_t^*(1+g)}{C_t^*}\left(\frac{1+r^*}{1+g} - \varphi_{KK}\right)$. Substituting in the quadratic equation (34) leads to a quadratic equation in $\varphi_{KK}$. Denote this quadratic equation as $f(\varphi_{KK}) = 0$, and note that $f(0) > 0$, $f(1) < 0$ and $\frac{\partial f(\varphi_{KK})^2}{\partial^2 \varphi_{KK}} > 0$. This implies that the quadratic equation $f(\varphi_{KK}) = 0$ has one solution between 0 and 1, and another solution which is greater than 1. To ensure stable dynamics,[10] we retain the solution for $\varphi_{KK}$ that is between 0 and 1; which means that of the two solutions for $\varphi_{CK}$ (given in equation (35)), we need to retain the largest one:

$$\varphi_{CK} = \frac{-Q_1 + \sqrt{Q_1^2 - 4Q_0Q_2}}{2Q_2} \tag{36}$$

Substituting in equations (32) and (33) yields then the solutions for $\varphi_{CA}$ and $\varphi_{CG}$:

$$\varphi_{CA} = \frac{\left[\varphi_{CK} + (1-\alpha)\frac{r^*+\delta}{1+r^*}\right](1-\alpha)\frac{Y^*}{K^*(1+g)} - (1-\alpha)\frac{r^*+\delta}{1+r^*}\phi_A}{1 + \left[\varphi_{CK} + (1-\alpha)\frac{r^*+\delta}{1+r^*}\right]\frac{C^*}{K^*(1+g)} - \phi_A} \tag{37}$$

$$\varphi_{CG} = -\frac{\left[\varphi_{CK} + (1-\alpha)\frac{r^*+\delta}{1+r^*}\right]\frac{G^*}{K^*(1+g)}}{1 + \left[\varphi_{CK} + (1-\alpha)\frac{r^*+\delta}{1+r^*}\right]\frac{C^*}{K^*(1+g)} - \phi_G} \tag{38}$$

We now have found all the $\varphi$-coefficients of equation (26), so we can compute $\hat{C}_t$ from period $t$'s state variables $\hat{K}_t$, $\hat{A}_t$ and $\hat{G}_t$. Once we know $\hat{C}_t$, the other endogenous variables can easily be found from equations (18), (19), (20), (21) and (23). The values of the state variables in period $t+1$ can be computed from equation (29), and equations (3) and (6) (moved one period forward).

# 6. Impulse response functions

We now calibrate the model by assigning appropriate values to $\alpha$, $\delta$, $\rho$, $A_t^*$, $G_t^*$, $\phi_A$, $\phi_G$, $g$ and $L$. Let us assume, for instance, that every period corresponds to a quarter, and let us choose parameter values that mimic the U.S. economy: $\alpha = 1/3$, $\delta = 2.5\%$, $\phi_A = 0.5$, $\phi_G = 0.5$, and $g = 0.5\%$; $A_t^*$ and $L$ are normalized to 1; $G_t^*$ is chosen such that $G_t^*/Y_t^* = 20\%$; and $\rho$ is chosen such that $r^* = 1.5\%$.[11]

It is then straightforward to compute the balanced growth path: $Y_t^* = 2.9$, $K_t^* = 24.1$, $I_t^* = 0.7$, $C_t^* = 1.6$ and $w_t^* = 1.9$ (while $r^* = 1.5\%$ per construction). $Y^*$, $K^*$, $I^*$, $C^*$ and $w^*$ all grow at rate 0.5% per quarter, while $r^*$ remains constant over time. Note that this parameterization yields an annual capital-output-ratio of about 2, while $C$ and $I$ are about 55% and 25% of $Y$, respectively - which seem reasonable numbers. Once we have computed the steady state, we can use equations (36), (37) and (38) to compute the $\varphi$-coefficients. We are then ready to trace out the economy's reaction to shocks in $A$ and $G$.

Consider first the effect of a technology shock in quarter 1. Suppose the economy is initially moving along its balanced growth path (such that $\hat{K}_s = \hat{A}_s = \hat{G}_s = 0$ $\forall s < 1$), when in quarter 1 it is suddenly hit by a technology shock $\varepsilon_{A,1} = 1$. From equation (3) follows then that $\hat{A}_1 = 1$ as well, while equations (29) and (6) imply that $\hat{K}_1 = \hat{G}_1 = 0$. Given these values for quarter 1's state variables and given the $\varphi$-coefficients, $\hat{C}_1$ can be computed from equation (26); the other endogenous variables in quarter 1 follow from equations (18), (19), (20), (21) and (23). Quarter 2's state variables can then be computed from equations (28), (3) and (6) - which leads to the values for quarter 2's endogenous variables, and quarter 3's state variables. In this way, we can trace out the effect of the technology shock into the infinite future.

**Figure 1: Effect of a 1% shock in $A$ ...**

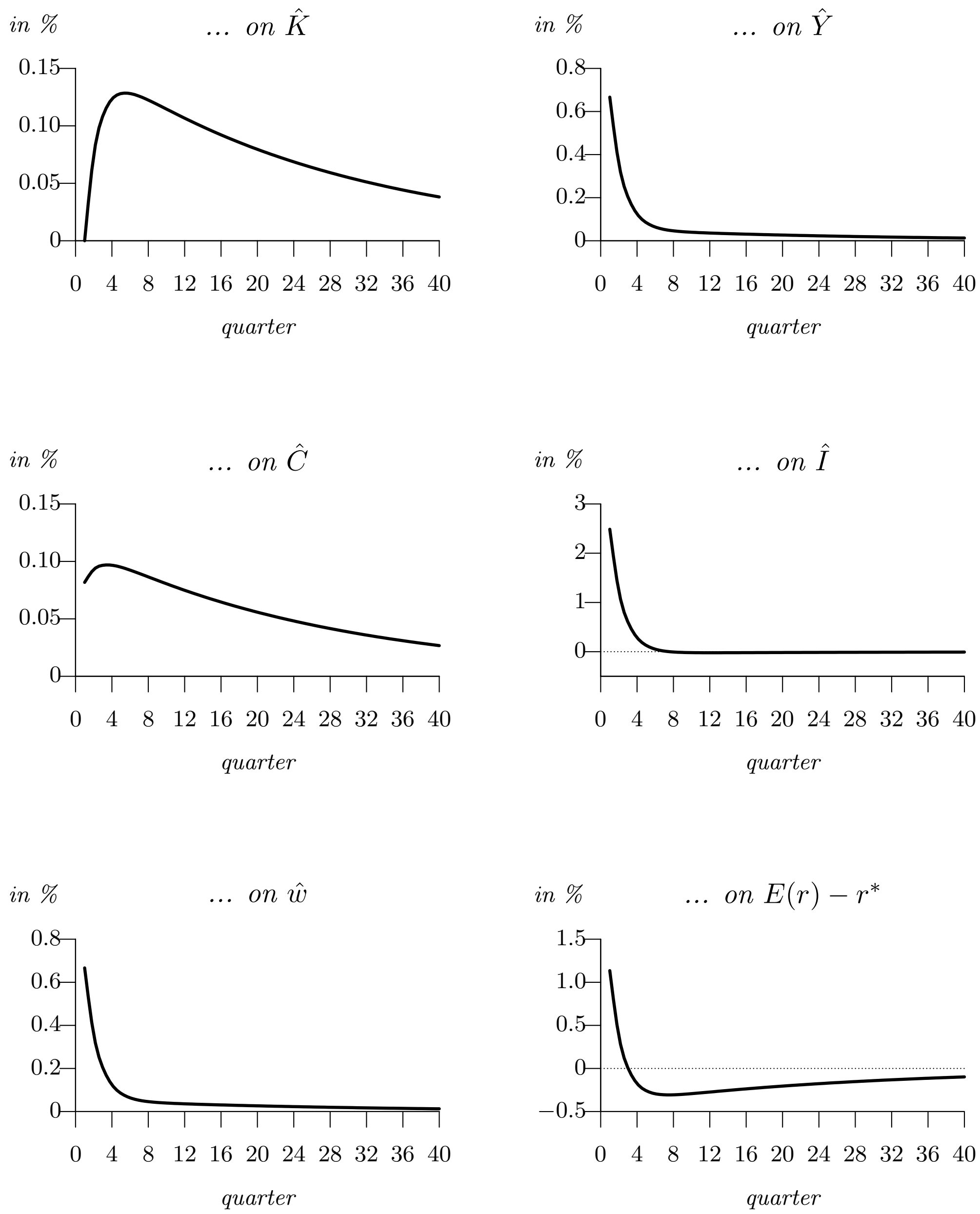

**Figure 2: Effect of a 1% shock in $G$ ...**

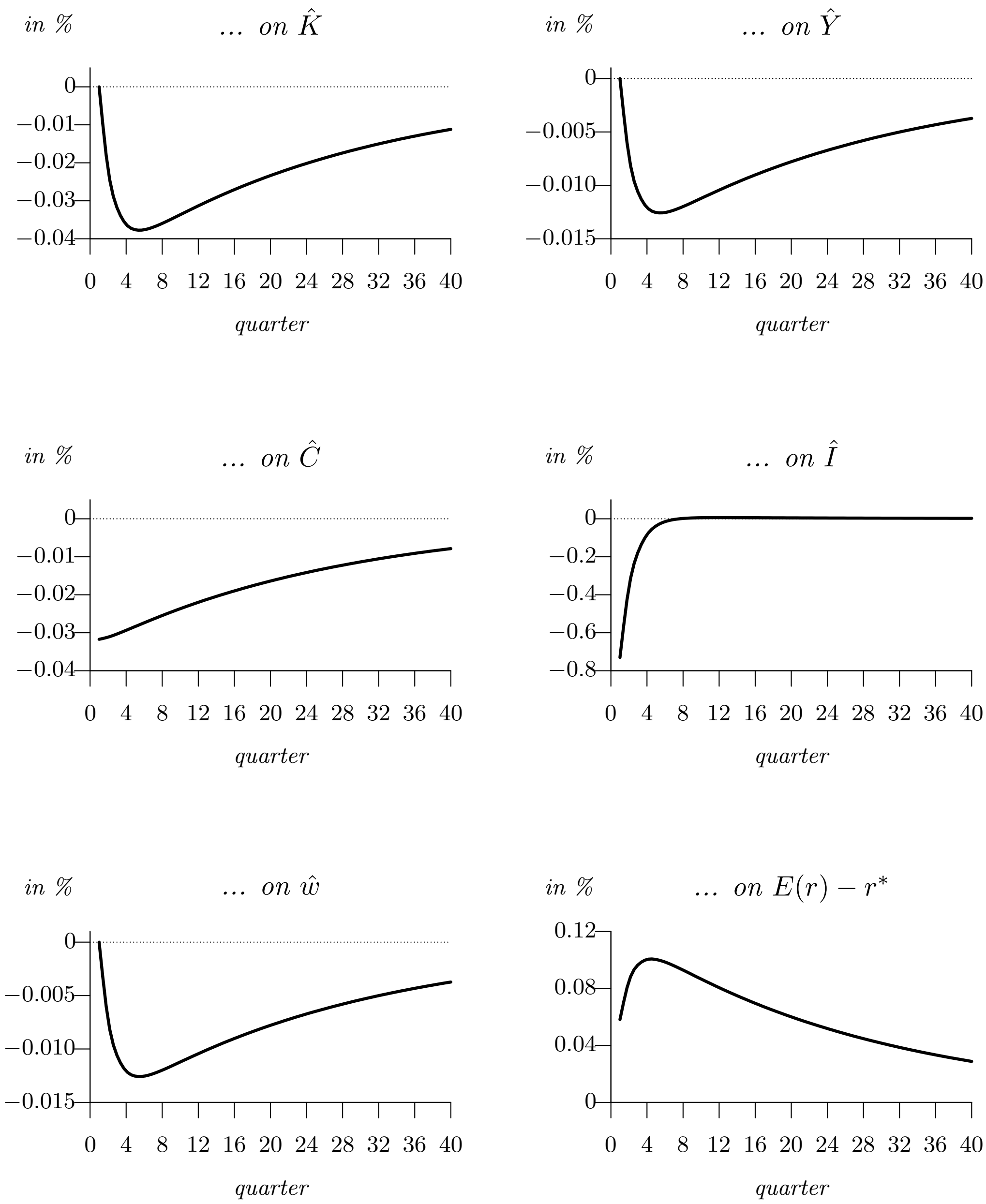

Figure 1 shows how the economy reacts during the first 40 quarters. Note that $Y$ jumps up in quarter 1, together with the technology shock. As a result, the representative household increases her consumption, but as she wants to smooth her consumption over time, $C$ increases less than $Y$. Investment $I$ therefore initially increases more than $Y$. As $I$ increases, the capital stock $K$ gradually increases as well after period 1. The expected rate of return, $E(r)$, is at first higher than on the balanced growth path (thanks to the technology shock). However, as the technology shock dies out while the capital stock builds up, the expected interest rate rapidly falls and even becomes negative after a few quarters. The real wage $w$ follows the time path of $Y$. Note that all variables eventually converge back to their steady state values.

Consider now the effect of a shock in government expenditures in quarter 1. Assume again that the economy is on a balanced growth path in quarter 0. In quarter 1, however, the economy is hit by a shock in government expenditures $\varepsilon_{G,1} = 1$. From equation (3) follows then that $\hat{A}_1 = 1$ as well, while equations (29) and (6) imply that $\hat{K}_1 = \hat{G}_1 = 0$. Once we know the state variables in quarter 1, we can compute the endogenous variables in quarter 1 and the state variables for quarter 2 in the same way as in the case of a technology shock - which leads to the values for quarter 2's endogenous variables and quarter 3's state variables, and so on until the infinite future.

Figure 2 shows the economy's reaction to a shock in government expenditures during the first 40 quarters. As $G$ increases, $E(r)$ increases as well such that $C$ and $I$ fall (to make sure that $C+I+G$ remains equal to $Y$, which does not change in quarter 1 as $\hat{K}_1 = 0$). As $I$ falls, the capital stock $K$ gradually decreases after period 1, such that $Y$ starts decreasing after period 1 as well. In the meantime, however, the shock in $G$ is dying out, so after a while $E(r)$ decreases again. As a result, $C$ and $I$ recover - and as $I$ recovers, $K$ and $Y$ recover also. Note that the real wage $w$ again follows the time path of $Y$. Eventually, all variables converge back to their steady state values.

# 7. Conclusions

This note presented the stochastic growth model, and solved the model by first linearizing it around a steady state and by then solving the linearized model with the method of undetermined coefficients.

Even though the stochastic growth model itself might bear little resemblance to the real world, it has proven to be a useful framework that can easily be extended to account for a wide range of macroeconomic issues that are potentially important. Kydland and Prescott (1982) introduced labor/leisure-substitution in the stochastic growth model, which gave rise to the so-called real-business-cycle literature. Greenwood and Huffman (1991) and Baxter and King (1993) replaced the lump-sum taxation by distortionary taxation, to study how taxes affect the behavior of firms and households. In the beginning of the 1990s, researchers started introducing money and nominal rigidities in the model, which gave rise to New Keynesian stochastic dynamic general equilibrium models that are now widely used to study monetary policy - see Goodfriend and King (1997) for an overview. Vermeylen (2006) shows how the representative household can be replaced by a large number of households to study the effect of job insecurity on consumption and saving in a general equilibrium setting.

[1] Microfoundations means that the objectives of the economic agents are formulated explicitly, and that their behavior is derived by assuming that they always try to achieve their objectives as well as they can.

[2] A steady state is a condition in which a number of key variables are not changing. In the stochastic growth model, these key variables are for instance the growth rate of aggregate production, the interest rate and the capital-output-ratio.

[3] See appendix A for derivations.

[4] This means that the present discounted value of public debt in the distant future should be equal to zero, such that public debt cannot keep on rising at a rate that is higher than the interest rate. This guarantees that public debt is always equal to the present discounted value of the government's future primary surpluses.

[5] Lump-sum taxes do not affect the first-order conditions of the firms and the households, and therefore do not affect their behavior either.

[6] Ricardian equivalence is the phenomenon that - given certain assumptions - it turns out to be irrelevant whether the government finances its expenditures by issuing public debt or by raising taxes. The reason for this is that given the time path of government expenditures, every increase in public debt must sooner or later be matched by an increase in taxes, such that the present discounted value of the taxes which a representative household has to pay is not affected by the way how the government finances its expenditures - which implies that her current wealth and her consumption path are not affected either.

[7] See appendix A for the derivation.

[8] See appendix B for the derivation.

[9] See appendix C for the derivations.

[10] The solution with unstable dynamics not only does not make sense from an economic point of view, it also violates the transversality conditions.

[11] Note that these values imply that the annual depreciation rate, the annual growth rate and the annual interest rate are about 10%, 2% and 6%, respectively.

# Appendix A

## A1. The maximization problem of the representative firm

The maximization problem of the firm can be rewritten as:

$$V_t(K_t) = \max_{\{L_t, I_t\}} \left\{ Y_t - w_t L_t - I_t + E_t \left[ \frac{1}{1+r_{t+1}} V_{t+1}(K_{t+1}) \right] \right\} \quad \text{(A.1)}$$

$$s.t. \quad Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha}$$
$$K_{t+1} = (1-\delta)K_t + I_t$$

The first-order conditions for $L_t$, respectively $I_t$, are:

$$(1-\alpha)K_t^{\alpha} A_t^{1-\alpha} L_t^{-\alpha} - w_t = 0 \quad \text{(A.2)}$$

$$-1 + E_t \left[ \frac{1}{1+r_{t+1}} \frac{\partial V_{t+1}(K_{t+1})}{\partial K_{t+1}} \right] = 0 \quad \text{(A.3)}$$

In addition, the envelope theorem implies that

$$\frac{\partial V_t(K_t)}{\partial K_t} = \alpha K_t^{\alpha-1}(A_t L_t)^{1-\alpha} + E_t \left[ \frac{1}{1+r_{t+1}} \frac{\partial V_{t+1}(K_{t+1})}{\partial K_{t+1}} \right] (1-\delta) \quad \text{(A.4)}$$

Substituting the production function in (A.2) gives equation (4):

$$(1-\alpha)\frac{Y_t}{L_t} = w_t$$

Substituting (A.3) in (A.4) yields:

$$\frac{\partial V_t(K_t)}{\partial K_t} = \alpha K_t^{\alpha-1}(A_t L_t)^{1-\alpha} + (1-\delta)$$

Moving one period forward, and substituting again in (A.3) gives:

$$-1 + E_t \left[ \frac{1}{1+r_{t+1}} \left( \alpha K_{t+1}^{\alpha-1}(A_{t+1} L_{t+1})^{1-\alpha} + (1-\delta) \right) \right] = 0$$

Substituting the production function in the equation above and reshuffling leads to equation (5):

$$1 = E_t \left[ \frac{1}{1+r_{t+1}} \alpha \frac{Y_{t+1}}{K_{t+1}} \right] + E_t \left[ \frac{1-\delta}{1+r_{t+1}} \right]$$

## A2. The maximization problem of the representative household

The maximization problem of the household can be rewritten as:

$$U_t(X_t) = \max_{\{C_t\}} \left\{ \ln C_t + \frac{1}{1+\rho} E_t \left[ U_{t+1}(X_{t+1}) \right] \right\} \quad \text{(A.5)}$$

$$s.t. \quad X_{t+1} = X_t(1+r_t) + w_t L - T_t - C_t$$

The first-order condition for $C_t$ is:

$$\frac{1}{C_t} - \frac{1}{1+\rho} E_t \left[ \frac{\partial U_{t+1}(X_{t+1})}{\partial X_{t+1}} \right] = 0 \tag{A.6}$$

In addition, the envelope theorem implies that

$$\frac{\partial U_t(X_t)}{\partial X_t} = \frac{1}{1+\rho} E_t \left[ \frac{\partial U_{t+1}(X_{t+1})}{\partial X_{t+1}} (1+r_t) \right] \tag{A.7}$$

Substituting (A.6) in (A.7) yields:

$$\frac{\partial U_t(X_t)}{\partial X_t} = (1+r_t) \frac{1}{C_t}$$

Moving one period forward, and substituting again in (A.6) gives the Euler equation (10):

$$\frac{1}{C_t} - E_t \left[ \frac{1+r_{t+1}}{1+\rho} \frac{1}{C_{t+1}} \right] = 0$$

# Appendix B

If $C$ grows at rate $g$, the Euler equation (10) implies that

$$C_s^*(1+g) \quad = \quad \frac{1+r^*}{1+\rho} C_s^*$$

Rearranging gives then the gross real rate of return $1+r^*$:

$$1+r^* = (1+g)(1+\rho)$$

which immediately leads to equation (17).

Subsituting in the firm's first-order condition (5) gives:

$$\alpha \frac{Y_{t+1}^*}{K_{t+1}^*} \quad = \quad r^* + \delta$$

Using the production function (1) to eliminate $Y$ yields:

$$\alpha K_{t+1}^{*\alpha-1}(A_{t+1}L)^{1-\alpha} \quad = \quad r^* + \delta$$

Rearranging gives then the value of $K_{t+1}^*$:

$$K_{t+1}^* \quad = \quad \left(\frac{\alpha}{r^*+\delta}\right)^{\frac{1}{1-\alpha}} A_{t+1}L$$

which is equivalent to equation (13).

Substituting in the production function (1) gives then equation (12):

$$Y_t^* = \left(\frac{\alpha}{r^* + \delta}\right)^{\frac{\alpha}{1-\alpha}} A_t L$$

Substituting (12) in the first-order condition (4) gives equation (16):

$$w_t^* = (1-\alpha)\left(\frac{\alpha}{r^* + \delta}\right)^{\frac{\alpha}{1-\alpha}} A_t$$

Substituting (13) in the law of motion (2) yields:

$$\left(\frac{\alpha}{r^* + \delta}\right)^{\frac{1}{1-\alpha}} A_{t+1} L = (1-\delta)\left(\frac{\alpha}{r^* + \delta}\right)^{\frac{1}{1-\alpha}} A_t L + I_t^*$$

such that $I_t^*$ is given by:

$$\begin{aligned} I_t^* &= \left(\frac{\alpha}{r^* + \delta}\right)^{\frac{1}{1-\alpha}} A_{t+1} L - (1-\delta)\left(\frac{\alpha}{r^* + \delta}\right)^{\frac{1}{1-\alpha}} A_t L \\ &= \left(\frac{\alpha}{r^* + \delta}\right)^{\frac{1}{1-\alpha}} [(1+g) - (1-\delta)]\, A_t L \\ &= (g+\delta)\left(\frac{\alpha}{r^* + \delta}\right)^{\frac{1}{1-\alpha}} A_t L \end{aligned}$$

...which is equation (14).

Consumption $C^*$ can then be computed from the equilibrium condition in the goods market:

$$\begin{aligned} C_t^* &= Y_t^* - I_t^* - G_t^* \\ &= \left(\frac{\alpha}{r^* + \delta}\right)^{\frac{\alpha}{1-\alpha}} A_t L - (g+\delta)\left(\frac{\alpha}{r^* + \delta}\right)^{\frac{1}{1-\alpha}} A_t L - G_t^* \\ &= \left[1 - \alpha\frac{g+\delta}{r^* + \delta}\right]\left(\frac{\alpha}{r^* + \delta}\right)^{\frac{\alpha}{1-\alpha}} A_t L - G_t^* \end{aligned}$$

Now recall that on the balanced growth path, $A$ and $G$ grow at the rate of technological progress $g$. The equation above then implies that $C^*$ also grows at the rate $g$, such that our initial educated guess turns out to be correct.

# Appendix C

## C1. The linearized production function

The production function is given by equation (1):

$$Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha}$$

Taking logarithms of both sides of this equation, and subtracting from both sides their values on the balanced growth path (taking into account that $\hat{L}_t = 0$), immediately yields the linearized version of the production function:

$$\begin{aligned}
\ln Y_t &= \alpha \ln K_t + (1-\alpha)\ln A_t + (1-\alpha)\ln L_t \\
\ln Y_t - \ln Y_t^* &= \alpha(\ln K_t - \ln K_t^*) + (1-\alpha)(\ln A_t - \ln A_t^*) + (1-\alpha)(\ln L_t - \ln L_t^*) \\
\hat{Y}_t &= \alpha \hat{K}_t + (1-\alpha)\hat{A}_t
\end{aligned}$$

...which is equation (18).

## C2. The linearized law of motion of the capital stock

The law of motion of the capital stock is given by equation (2):

$$K_{t+1} = (1-\delta)K_t + I_t$$

Taking logarithms of both sides of this equation, and subtracting from both sides their values on the balanced growth path, yields:

$$\ln K_{t+1} - \ln K^*_{t+1} = \ln\{(1-\delta)K_t + I_t\} - \ln K^*_{t+1}$$

Now take a first-order Taylor-approximation of the right-hand-side around $\ln K_t = \ln K^*_t$ and $\ln I_t = \ln I^*_t$:

$$\begin{aligned} \ln K_{t+1} - \ln K^*_{t+1} &= \varphi_1(\ln K_t - \ln K^*_t) + \varphi_2(\ln I_t - \ln I^*_t) \\ \hat{K}_{t+1} &= \varphi_1 \hat{K}_t + \varphi_2 \hat{I}_t \end{aligned} \tag{C.1}$$

where

$$\begin{aligned} \varphi_1 &= \left(\frac{\partial \ln\{(1-\delta)K_t + I_t\}}{\partial \ln K_t}\right)^* \\ \varphi_2 &= \left(\frac{\partial \ln\{(1-\delta)K_t + I_t\}}{\partial \ln I_t}\right)^* \end{aligned}$$

$\varphi_1$ and $\varphi_2$ can be worked out as follows:

$$\begin{aligned} \varphi_1 &= \left(\frac{\partial \ln\{(1-\delta)K_t + I_t\}}{\partial K_t}\frac{\partial K_t}{\partial \ln K_t}\right)^* \\ &= \left(\frac{1-\delta}{(1-\delta)K_t + I_t}K_t\right)^* \\ &= \left(\frac{1-\delta}{K_{t+1}}K_t\right)^* \\ &= \frac{1-\delta}{1+g} \qquad \text{...as } K_t \text{ grows at rate } g \text{ on the balanced growth path} \\ \varphi_2 &= \left(\frac{\partial \ln\{(1-\delta)K_t + I_t\}}{\partial I_t}\frac{\partial I_t}{\partial \ln I_t}\right)^* \end{aligned}$$

$$\begin{aligned} &= \left(\frac{1}{(1-\delta)K_t + I_t}I_t\right)^* \\ &= \left(\frac{1}{K_{t+1}}I_t\right)^* \\ &= \frac{g+\delta}{1+g} \qquad \text{...as } I^*_t/K^*_t = g+\delta \text{ and } K_t \text{ grows at rate } g \text{ on the balanced growth path} \end{aligned}$$

Substituting in equation (C.1) gives then the linearized law of motion for $K$:

$$\hat{K}_{t+1} = \frac{1-\delta}{1+g}\hat{K}_t + \frac{g+\delta}{1+g}\hat{I}_t$$

...which is equation (19).

## C3. The linearized first-order condotion for the firm's labor demand

The first-order condition for the firm's labor demand is given by equation (4):

$$(1-\alpha)\frac{Y_t}{L_t} = w_t$$

Taking logarithms of both sides of this equation, and subtracting from both sides their values on the balanced growth path (taking into account that $\hat{L}_t = 0$), immediately yields the linearized version of this first-order condition:

$$\begin{aligned}
\ln(1-\alpha) + \ln Y_t - \ln L_t &= \ln w_t \\
(\ln Y_t - \ln Y_t^*) - (\ln L_t - \ln L^*) &= \ln w_t - \ln w_t^* \\
\hat{Y}_t &= \hat{w}_t
\end{aligned}$$

...which is equation (20).

## C4. The linearized first-order condotion for the firm's capital demand

The first-order condition for the firm's capital demand is given by equation (5):

$$1 = E_t[Z_{t+1}] \tag{C.2}$$

$$\text{with } Z_{t+1} = \tfrac{1}{1+r_{t+1}}\alpha\tfrac{Y_{t+1}}{K_{t+1}} + \tfrac{1-\delta}{1+r_{t+1}} \tag{C.3}$$

Now take a first-order Taylor-approximation of the right-hand-side of equation (C.3) around $\ln Y_{t+1} = \ln Y_{t+1}^*$, $\ln K_{t+1} = \ln K_{t+1}^*$ and $r_{t+1} = r^*$:

$$\begin{aligned}
Z_{t+1} &= 1 + \varphi_1(\ln Y_{t+1} - \ln Y_{t+1}^*) + \varphi_2(\ln K_{t+1} - \ln K_{t+1}^*) + \varphi_3(r_{t+1} - r^*) \\
&= 1 + \varphi_1\hat{Y}_{t+1} + \varphi_2\hat{K}_{t+1} + \varphi_3(r_{t+1} - r^*) \qquad (C.4)
\end{aligned}$$

where

$$\begin{aligned}
\varphi_1 &= \left(\frac{\partial\left\{\frac{1}{1+r_{t+1}}\alpha\frac{Y_{t+1}}{K_{t+1}}+\frac{1-\delta}{1+r_{t+1}}\right\}}{\partial \ln Y_{t+1}}\right)^* \\
\varphi_2 &= \left(\frac{\partial\left\{\frac{1}{1+r_{t+1}}\alpha\frac{Y_{t+1}}{K_{t+1}}+\frac{1-\delta}{1+r_{t+1}}\right\}}{\partial \ln K_{t+1}}\right)^* \\
\varphi_3 &= \left(\frac{\partial\left\{\frac{1}{1+r_{t+1}}\alpha\frac{Y_{t+1}}{K_{t+1}}+\frac{1-\delta}{1+r_{t+1}}\right\}}{\partial r_{t+1}}\right)^*
\end{aligned}$$

$\varphi_1$, $\varphi_2$ and $\varphi_3$ can be worked out as follows:

$$\begin{aligned}
\varphi_1 &= \left(\frac{\partial\left\{\frac{1}{1+r_{t+1}}\alpha\frac{Y_{t+1}}{K_{t+1}}+\frac{1-\delta}{1+r_{t+1}}\right\}}{\partial Y_{t+1}}\frac{\partial Y_{t+1}}{\partial \ln Y_{t+1}}\right)^* \\
&= \left(\frac{1}{1+r_{t+1}}\alpha\frac{1}{K_{t+1}}Y_{t+1}\right)^* \\
&= \frac{r^*+\delta}{1+r^*} \quad \text{...using the fact that } \alpha Y_{t+1}^* = (r^*+\delta)K_{t+1}^*
\end{aligned}$$

$$\begin{aligned}
\varphi_2 &= \left(\frac{\partial\left\{\frac{1}{1+r_{t+1}}\alpha\frac{Y_{t+1}}{K_{t+1}}+\frac{1-\delta}{1+r_{t+1}}\right\}}{\partial K_{t+1}}\frac{\partial K_{t+1}}{\partial \ln K_{t+1}}\right)^* \\
&= -\left(\frac{1}{1+r_{t+1}}\alpha\frac{Y_{t+1}}{K_{t+1}^2}K_{t+1}\right)^* \\
&= -\frac{r^*+\delta}{1+r^*} \quad \text{...using the fact that } \alpha Y_{t+1}^* = (r^*+\delta)K_{t+1}^*
\end{aligned}$$

$$\begin{aligned}
\varphi_3 &= -\left(\frac{1}{(1+r_{t+1})^2}\left[\alpha\frac{Y_{t+1}}{K_{t+1}}+1-\delta\right]\right)^* \\
&= -\frac{1}{1+r^*} \qquad \text{(C.5)}
\end{aligned}$$

Substituting in equation (C.4) gives then:

$$Z_{t+1} = 1+\frac{r^*+\delta}{1+r^*}\hat{Y}_{t+1}-\frac{r^*+\delta}{1+r^*}\hat{K}_{t+1}-\frac{r_{t+1}-r^*}{1+r^*}$$

Substituting in equation (C.2) and rearranging, gives then equation (21):

$$E_t\left[\frac{r_{t+1}-r^*}{1+r^*}\right] = \frac{r^*+\delta}{1+r^*}\left[E_t(\hat{Y}_{t+1})-E_t(\hat{K}_{t+1})\right]$$

## C5. The linearized Euler equation of the representative household

The Euler equation of the representative household is given by equation (10), which is equivalent to:

$$1 = E_t[Z_{t+1}] \tag{C.6}$$
$$\text{with } Z_{t+1} = \tfrac{1+r_{t+1}}{1+\rho}\tfrac{C_t}{C_{t+1}} \tag{C.7}$$

Now take a first-order Taylor-approximation of the right-hand-side of equation (C.7) around $\ln C_{t+1} = \ln C^*_{t+1}$, $\ln C_t = \ln C^*_t$ and $r_{t+1} = r^*$:

$$\begin{aligned} Z_{t+1} &= 1 + \varphi_1(\ln C_{t+1} - \ln C^*_{t+1}) + \varphi_2(\ln C_t - \ln C^*_t) + \varphi_3(r_{t+1} - r^*) \\ &= 1 + \varphi_1\hat{C}_{t+1} + \varphi_2\hat{C}_t + \varphi_3(r_{t+1} - r^*) \end{aligned} \tag{C.8}$$

where

$$\begin{aligned} \varphi_1 &= \left(\frac{\partial\left\{\frac{1+r_{t+1}}{1+\rho}\frac{C_t}{C_{t+1}}\right\}}{\partial \ln C_{t+1}}\right)^* \\ \varphi_2 &= \left(\frac{\partial\left\{\frac{1+r_{t+1}}{1+\rho}\frac{C_t}{C_{t+1}}\right\}}{\partial \ln C_t}\right)^* \\ \varphi_3 &= \left(\frac{\partial\left\{\frac{1+r_{t+1}}{1+\rho}\frac{C_t}{C_{t+1}}\right\}}{\partial r_{t+1}}\right)^* \end{aligned}$$

$\varphi_1$, $\varphi_2$ and $\varphi_3$ can be worked out as follows:

$$\begin{aligned} \varphi_1 &= \left(\frac{\partial\left\{\frac{1+r_{t+1}}{1+\rho}\frac{C_t}{C_{t+1}}\right\}}{\partial C_{t+1}}\frac{\partial C_{t+1}}{\partial \ln C_{t+1}}\right)^* \\ &= -\left(\frac{1+r_{t+1}}{1+\rho}\frac{C_t}{C^2_{t+1}}C_{t+1}\right)^* \\ &= -1 \end{aligned}$$

$$\begin{aligned} \varphi_2 &= \left(\frac{\partial\left\{\frac{1+r_{t+1}}{1+\rho}\frac{C_t}{C_{t+1}}\right\}}{\partial C_t}\frac{\partial C_t}{\partial \ln C_t}\right)^* \\ &= \left(\frac{1+r_{t+1}}{1+\rho}\frac{1}{C_{t+1}}C_t\right)^* \\ &= 1 \end{aligned}$$

$$\begin{aligned} \varphi_3 &= \left(\frac{1}{1+\rho}\frac{C_t}{C_{t+1}}\right)^* \\ &= \left(\frac{\frac{1+r_{t+1}}{1+\rho}\frac{C_t}{C_{t+1}}}{1+r_{t+1}}\right)^* \end{aligned}$$

$$= \frac{1}{1+r^*}$$

Substituting in equation (C.8) gives then:

$$Z_{t+1} = 1 - \hat{C}_{t+1} + \hat{C}_t + \frac{r_{t+1} - r^*}{1+r^*}$$

Substituting in equation (C.6) and rearranging, gives then equation (22):

$$\hat{C}_t = E_t\left[\hat{C}_{t+1}\right] - E_t\left[\frac{r_{t+1} - r^*}{1+r^*}\right]$$

# C6. The linearized equillibrium condition in the goods market

The equilibrium condition in the goods market is given by equation (11):

$$Y_t = C_t + I_t + G_t$$

Taking logarithms of both sides of this equation, and subtracting from both sides their values on the balanced growth path, yields:

$$\ln Y_t - \ln Y_t^* = \ln\left(C_t + I_t + G_t\right) - \ln Y_t^*$$

Now take a first-order Taylor-approximation of the right-hand-side around $\ln C_t = \ln C_t^*$, $\ln I_t = \ln I_t^*$ and $\ln G_t = \ln G_t^*$:

$$\begin{aligned} \ln Y_t - \ln Y_t^* &= \varphi_1(\ln C_t - \ln C_t^*) + \varphi_2(\ln I_t - \ln I_t^*) + \varphi_3(\ln G_t - \ln G_t^*) \\ \hat{Y}_t &= \varphi_1 \hat{C}_t + \varphi_2 \hat{I}_t + \varphi_3 \hat{G}_t \end{aligned} \tag{C.9}$$

where

$$\begin{aligned} \varphi_1 &= \left(\frac{\partial \ln\{C_t + I_t + G_t\}}{\partial \ln C_t}\right)^* \\ \varphi_2 &= \left(\frac{\partial \ln\{C_t + I_t + G_t\}}{\partial \ln I_t}\right)^* \\ \varphi_3 &= \left(\frac{\partial \ln\{C_t + I_t + G_t\}}{\partial \ln G_t}\right)^* \end{aligned}$$

$\varphi_1$, $\varphi_2$ and $\varphi_3$ can be worked out as follows:

$$\begin{aligned} \varphi_1 &= \left(\frac{\partial \ln\{C_t + I_t + G_t\}}{\partial C_t} \frac{\partial C_t}{\partial \ln C_t}\right)^* \\ &= \left(\frac{1}{C_t + I_t + G_t} C_t\right)^* \\ &= \frac{C_t^*}{Y_t^*} \end{aligned}$$

$$
\begin{aligned}
\varphi_2 &= \left(\frac{\partial \ln\{C_t + I_t + G_t\}}{\partial I_t}\frac{\partial I_t}{\partial \ln I_t}\right)^* \\
&= \left(\frac{1}{C_t + I_t + G_t} I_t\right)^* \\
&= \frac{I_t^*}{Y_t^*} \\
\varphi_3 &= \left(\frac{\partial \ln\{C_t + I_t + G_t\}}{\partial G_t}\frac{\partial G_t}{\partial \ln G_t}\right)^* \\
&= \left(\frac{1}{C_t + I_t + G_t} G_t\right)^* \\
&= \frac{G_t^*}{Y_t^*}
\end{aligned}
$$

Substituting in equation (C.9) gives then the linearized equilibrium condition in the goods market:

$$
\hat{Y}_t = \frac{C_t^*}{Y_t^*}\hat{C}_t + \frac{I_t^*}{Y_t^*}\hat{I}_t + \frac{G_t^*}{Y_t^*}\hat{G}_t
$$

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